

Penguin Effects in $\phi_{d,s}$ Determinations

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Abstract: *The theoretical precision of the measurement of the $B_{d,s}^0-\bar{B}_{d,s}^0$ mixing phases $\phi_{d,s}$ through the benchmark decays $B_d^0 \rightarrow J/\psi K_S$, $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi f_0(980)$ is limited by doubly Cabibbo-suppressed penguin topologies which are usually neglected. However, the search for New-Physics effects in the quark-flavor sector has entered a territory where these effects have to be taken into account, which will be particularly relevant for the LHCb upgrade era. Thanks to their non-perturbative nature, the penguin corrections cannot be calculated but have rather to be controlled through experimental data. An overview of the picture of the penguin parameters originating from the current data for $B_{(s)} \rightarrow J/\psi \pi, J/\psi K$ decays and the physics potential of the control channels $B_s^0 \rightarrow J/\psi K_S$, $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$ and $B_d^0 \rightarrow J/\psi f_0(980)$ is given, emphasizing also the usefulness of effective B_s decay lifetimes.*

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1 Introduction

This summer, we have received the great news that a Higgs-like particle was discovered by the ATLAS and CMS collaborations at the LHC. On the other hand, these experiments could still not reveal any deviation from the Standard Model (SM) at the high-energy frontier, while the LHCb experiment operating at the high-precision frontier could also not yet resolve evidence for New Physics (NP) in the quark-flavor sector. Concerning the structure of possible physics lying beyond the SM, we have therefore to deal with a larger characteristic NP scale Λ_{NP} , i.e. not just $\Lambda_{\text{NP}} \sim \text{TeV}$, or/and symmetries prevent large NP effects in the flavor sector, where the most prominent example is given by models with “Minimal Flavor Violation”.

Many more results are yet to come, but in view of the current situation we have to prepare ourselves to deal with smallish NP effects. In order to resolve such phenomena, it is crucial to have a critical look at theoretical analyses and the approximations involved. The central issue is related to strong interactions and “hadronic” uncertainties. In particular, the theoretical and experimental precisions have to be matched to one another, which will be especially relevant for the LHCb upgrade program.

Concerning the further exploration of CP violation, $B_d \rightarrow J/\psi K_S$, $B_s \rightarrow J/\psi \phi$ and $B_s \rightarrow J/\psi f_0(980)$ decays play outstanding roles, allowing measurements of the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing phases

$$\phi_d = 2\beta + \phi_d^{\text{NP}}, \quad \phi_s = -2\delta\gamma + \phi_s^{\text{NP}}. \quad (1)$$

Here the former pieces are the SM contributions $\phi_q^{\text{SM}} = 2\arg(V_{tq}^* V_{tb})$, with β denoting the usual angle of the CKM unitarity triangle, and $\delta\gamma \approx 1^\circ$ [1]. From the theoretical point of view, these measurements are affected by uncertainties from doubly Cabibbo-suppressed penguin contributions [2]–[8]. These effects are usually neglected in the experimental analyses and are naively expected to be very small. However, as they are related to non-perturbative long-distance dynamics, the corresponding parameters cannot be calculated within perturbative QCD. Consequently, the question arises how big these effects are and how they can be controlled by means of experimental data.

2 $B_d^0 \rightarrow J/\psi K_S$ and $B_s^0 \rightarrow J/\psi K_S$

In the SM, the $B_d^0 \rightarrow J/\psi K_S$ decay amplitude can be written as follows [2]:

$$A(B_d^0 \rightarrow J/\psi K_S) = (1 - \lambda^2/2) \mathcal{A}' (1 + \epsilon a' e^{i\theta'} e^{i\gamma}). \quad (2)$$

Here $\lambda \equiv |V_{us}|$ is the Wolfenstein parameter, γ denotes the usual angle of the CKM unitarity triangle, and the following CP-conserving hadronic parameters enter:

$$\mathcal{A}' \equiv \lambda^2 A [A_T^{(c)'} + A_P^{(c)'} - A_P^{(t)'}], \quad a' e^{i\theta'} \equiv R_b \left[\frac{A_P^{(u)'} - A_P^{(t)'}}{A_T^{(c)'} + A_P^{(c)'} - A_P^{(t)'}} \right], \quad (3)$$

where $A_T^{(c)'}$ is the color-suppressed tree contribution and the $A_P^{(q)'}$ denote penguin topologies with internal q quarks. The primes remind us that we are dealing with a $\bar{b} \rightarrow \bar{c} c \bar{s}$ transition. Moreover, the decay amplitude involves the CKM factors

$$A \equiv \frac{1}{\lambda^2} |V_{cb}| \sim 0.8, \quad R_b \equiv \left(1 - \frac{\lambda^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| \sim 0.5, \quad \epsilon \equiv \frac{\lambda^2}{1 - \lambda^2} = 0.053. \quad (4)$$

The parameters in (3) suffer from large hadronic uncertainties, in particular the $a' e^{i\theta'}$, which measures the ratio of tree to penguin contributions. However, as the latter quantity is doubly Cabibbo-suppressed in (2) by the tiny ϵ , it is usually neglected.

The $B_d \rightarrow J/\psi K_S$ channel offers the following time-dependent CP asymmetry:

$$\begin{aligned} & \frac{\Gamma(B_d^0(t) \rightarrow J/\psi K_S) - \Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S)}{\Gamma(B_d^0(t) \rightarrow J/\psi K_S) + \Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S)} \\ &= C(B_d \rightarrow J/\psi K_S) \cos(\Delta M_d t) - S(B_d \rightarrow J/\psi K_S) \sin(\Delta M_d t), \end{aligned} \quad (5)$$

where

$$C(B_d \rightarrow J/\psi K_S) = -\frac{2\epsilon a' \sin \theta' \sin \gamma}{1 + 2\epsilon a' \cos \theta' \cos \gamma + \epsilon^2 a'^2} \quad (6)$$

describes direct CP violation, and the “mixing-induced” CP asymmetry

$$\frac{S(B_d \rightarrow J/\psi K_S)}{\sqrt{1 - C(B_d \rightarrow J/\psi K_S)^2}} = \sin(\phi_d + \Delta\phi_d) \quad (7)$$

originates from the interference between B_d^0 - \bar{B}_d^0 mixing and decay processes. The phase shift $\Delta\phi_d$ is given by the following expression [4]:

$$\tan \Delta\phi_d = \frac{2\epsilon a' \cos \theta' \sin \gamma + \epsilon^2 a'^2 \sin 2\gamma}{1 + 2\epsilon a' \cos \theta' \cos \gamma + \epsilon^2 a'^2 \cos 2\gamma}. \quad (8)$$

The most recent Heavy Flavor Averaging Group (HFAG) compilation [9] gives

$$C(B_d \rightarrow J/\psi K_S) = 0.024 \pm 0.026 \Rightarrow \sqrt{1 - C(B_d \rightarrow J/\psi K_S)^2} = 0.9997_{-0.0010}^{+0.0003}, \quad (9)$$

so that (7) can be simplified with excellent precision as

$$S(B_d \rightarrow J/\psi K_S) = \sin(\phi_d + \Delta\phi_d) = 0.665 \pm 0.024. \quad (10)$$

This expression illustrates the theoretical limitation of the measurement of ϕ_d through the phase shift $\Delta\phi_d$, which is caused by the doubly Cabibbo-suppressed penguin contributions, as can be seen in (8). For values of $a' \sim 0.2$, the $\Delta\phi_d$ can be as large as about 1° , depending on the strong phase θ' .

How can we control $\Delta\phi_d$? As a' and θ' cannot be calculated reliably, we use the control channel $B_s^0 \rightarrow J/\psi K_S$, which is caused by $\bar{b} \rightarrow \bar{c}c\bar{d}$ quark-level processes and is related to $B_d^0 \rightarrow J/\psi K_S$ through the U -spin flavor symmetry of strong interactions [2]. Its decay amplitude can be written in the SM as

$$A(B_s^0 \rightarrow J/\psi K_S) = -\lambda \mathcal{A} (1 - ae^{i\theta} e^{i\gamma}), \quad (11)$$

where the unprimed amplitudes are defined in analogy to their counterparts in (3). The U -spin symmetry implies $a = a'$ and $\theta = \theta'$. The key feature of (11) is the absence of the ϵ suppression factor in front of the $ae^{i\theta}$. Consequently, the impact of this parameter is magnified in the corresponding observables.

As was pointed out in [2], γ as well as a and θ can be determined from the CP asymmetries of $B_s \rightarrow J/\psi K_S$ and the ratio of the $B_d \rightarrow J/\psi K_S$, $B_s \rightarrow J/\psi K_S$ branching ratios. While the γ determination appeared most interesting back in 1999, there has been a recent change of focus [6]: the extraction of γ looks feasible at the LHCb upgrade but probably not competitive with other methods. However, using γ as an input, the hadronic parameters a , θ can be determined in a *clean* way from the

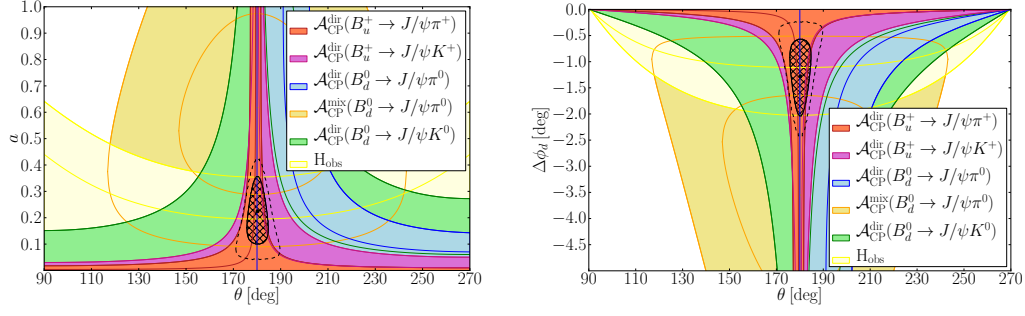


Figure 1: Current experimental constraints for the penguin parameters a , θ (left) and the phase shift $\Delta\phi_d$ (right), showing the 39% and 68% C.L. regions (from [13]).

CP-violating $B_s \rightarrow J/\psi K_S$ observables, thereby allowing us to get a handle on the penguin effects in the measurement of ϕ_d from $S(B_d \rightarrow J/\psi K_S)$.

The $B_s \rightarrow J/\psi K_S$ channel has been observed by CDF [10] and LHCb [11], but so far only measurements of the branching ratio are available, where subtleties related to the sizable B_s decay width difference $\Delta\Gamma_s$ have to be taken into account [12]. These branching ratio measurements are consistent with an $SU(3)$ relation to the branching ratio of $B_d \rightarrow J/\psi\pi^0$ [6, 11]. It is useful to introduce the ratio

$$H \equiv \frac{1}{\epsilon} \left| \frac{\mathcal{A}'}{\mathcal{A}} \right|^2 \left[\frac{\tau_{B_d} \Phi_{J/\psi K_S}^d}{\tau_{B_s} \Phi_{J/\psi K_S}^s} \right] \frac{\text{BR}(B_s \rightarrow J/\psi K_S)}{\text{BR}(B_d \rightarrow J/\psi K_S)}, \quad (12)$$

where the Φ and τ_{B_q} denote phase-space factors and B_q lifetimes, respectively.

In order to constrain a , θ from currently available data, we use also decays with a CKM structure similar to $B_s \rightarrow J/\psi K_S$, i.e. $B_d^0 \rightarrow J/\psi\pi^0$ and $B^+ \rightarrow J/\psi\pi^+$, and complement them with $B_d^0 \rightarrow J/\psi K^0$, $B^+ \rightarrow J/\psi K^+$ decays. These channels allow the construction of a variety of H ratios in analogy to (12). The data give an internally consistent picture, with the average $H_{\text{obs}} = 1.19 \pm 0.04(\text{stat}) \pm 0.21(\text{FF})$, which takes also $SU(3)$ -breaking corrections through form-factor (FF) ratios into account [13]. In Fig. 1, the current picture is shown, corresponding to the following 1σ ranges:

$$a = 0.22 \pm 0.13, \quad \theta = (180.2 \pm 4.5)^\circ, \quad \Delta\phi_d = -(1.28 \pm 0.74)^\circ. \quad (13)$$

The situation of the analysis and extraction of the penguin parameters for the LHCb upgrade looks promising [6, 13], where the $B_s \rightarrow J/\psi K_S$ channel is expected to play the role of a golden mode to explore the importance of penguin topologies.

3 $B_s^0 \rightarrow J/\psi\phi$ and $B_s^0 \rightarrow J/\psi\bar{K}^{*0}$

The decay $B_s^0 \rightarrow J/\psi\phi$ is the B_s -meson counterpart of the $B_d^0 \rightarrow J/\psi K_S$ channel, allowing the extraction of the B_s^0 - \bar{B}_s^0 mixing phase ϕ_s . Since the final state involves

two vector mesons, a time-dependent angular analysis has to be performed in order to disentangle the CP eigenstates [14]. In analogy to $B_d^0 \rightarrow J/\psi K_S$, the analysis of CP violation in the $B_s^0 \rightarrow J/\psi\phi$ channel is also affected by doubly Cabibbo-suppressed penguin contributions [5]. For a given final-state configuration $f \in \{0, \parallel, \perp\}$, the SM decay amplitude can be written as

$$A(B_s^0 \rightarrow (J/\psi\phi)_f) = (1 - \lambda^2/2) \mathcal{A}'_f [1 + \epsilon a'_f e^{i\theta'_f} e^{i\gamma}], \quad (14)$$

and the mixing-induced CP asymmetries take the form

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow (J/\psi\phi)_f) = \sin(\phi_s + \Delta\phi_s^f), \quad (15)$$

which is the counterpart of (10). In the literature, it is usually assumed that $\Delta\phi_s^f = 0$. The most recent average compiled by HFAG reads $\phi_s = -(0.74^{+5.2}_{-4.8})^\circ$ [9], whereas we have $\phi_s = -(2.08 \pm 0.09)^\circ$ in the SM [15]. In view of the small value of ϕ_s emerging from the data, a phase shift $\Delta\phi_s^f$ at the 1° level (see (13)) would have a significant impact for the resolution of possible NP effects.

A channel to probe these penguin contributions is offered by $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$, with a SM decay amplitude of the structure

$$A(B_s^0 \rightarrow (J/\psi \bar{K}^{*0})_f) = -\lambda \mathcal{A}_f [1 - a_f e^{i\theta_f} e^{i\gamma}], \quad (16)$$

which is similar to (11). In particular, $a_f e^{i\theta_f}$ is again not suppressed by the tiny ϵ parameter. Neglecting penguin annihilation (PA) and exchange topologies (E), which can be constrained by the upper bound on $\text{BR}(B_d \rightarrow J/\psi\phi)$ as $|E + PA|/|T| \lesssim 0.1$, and using the $SU(3)$ flavor symmetry, we get the relations $a_f = a'_f$ and $\theta_f = \theta'_f$, allowing us to determine/constrain the penguin shift $\Delta\phi_s^f$ in (15) [5].

In contrast to $B_s^0 \rightarrow J/\psi K_S$, $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$ is flavor-specific and does, hence, not show mixing-induced CP violation. Consequently, the implementation of this method has to use measurements of untagged and direct CP-violating observables, and an angular analysis is required to disentangle the final-state configurations f .

The $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$ decay was observed by CDF [10] and LHCb [16]. The most recent LHCb branching ratio $(4.4^{+0.5}_{-0.4} \pm 0.8) \times 10^{-5}$ agrees well with the prediction $(4.6 \pm 0.4) \times 10^{-5}$ obtained from the $\text{BR}(B_d \rightarrow J/\psi \rho^0)$ by means of the $SU(3)$ flavor symmetry [5], and the polarization fractions agree well with those of $B_d^0 \rightarrow J/\psi K^{*0}$.

The experimental sensitivity for the extraction of ϕ_s from $B_s \rightarrow J/\psi\phi$ at the LHCb upgrade (50 fb^{-1}) is expected as $\Delta\phi_s|_{\text{exp}} \sim 0.008 = 0.46^\circ$ [17]. In view of this impressive precision on the one hand and $\Delta\phi_d = -(1.28 \pm 0.74)^\circ$ following from the current data for $B \rightarrow J/\psi\pi, J/\psi K$ decays with a dynamics similar to $B_s \rightarrow J/\psi\phi$ (see Section 2) on the other hand, it will be crucial to get a handle on the penguin effects at the LHCb upgrade as they may mimic NP effects.

4 $B_s^0 \rightarrow J/\psi f_0(980)$ and $B_d^0 \rightarrow J/\psi f_0(980)$

Another decay that has recently entered the stage is $B_s^0 \rightarrow J/\psi f_0(980)$, which was observed by LHCb [18], Belle [19], CDF [20] and D0 [21]. The dominant decay mode is via $f_0 \rightarrow \pi^+\pi^-$, with a branching ratio about four times smaller than that of $B_s^0 \rightarrow J/\psi \phi$ with $\phi \rightarrow K^+K^-$. However, since the $f_0 \equiv f_0(980)$ is a scalar $J^{PC} = 0^{++}$ state no angular analysis is required, thereby simplifying the analysis considerably and offering an interesting alternative for the determination of ϕ_s [22].

The impact of hadronic uncertainties on the extraction of ϕ_s from CP violation in $B_s^0 \rightarrow J/\psi f_0$ was studied in detail in [7], and for the $B_{s,d} \rightarrow J/\psi \eta^{(\prime)}$ system in [23]. The general formalism is very similar to the discussion given above:

$$A(B_s^0 \rightarrow J/\psi f_0) \propto [1 + \epsilon b e^{i\vartheta} e^{i\gamma}], \quad (17)$$

i.e. the hadronic penguin corrections enter again in a doubly Cabibbo-suppressed way. The mixing-induced CP asymmetry can be written as

$$S(B_s^0 \rightarrow J/\psi f_0) = \sqrt{1 - C(B_s^0 \rightarrow J/\psi f_0)^2} \sin(\phi_s + \Delta\tilde{\phi}_s), \quad (18)$$

where $\Delta\tilde{\phi}_s$ is given by an expression analogous to (8). However, in contrast to the $B_d \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi \phi$ decays, the $B_s \rightarrow J/\psi f_0$ channel suffers from the fact that the hadronic structure of the $f_0(980)$ is poorly known: popular benchmark scenarios are the quark–antiquark and tetraquark pictures. In the latter case, a peculiar decay topology arises at the tree level that does not have a counterpart in the quark–antiquark description [7].

The parameter b depends on the hadronic composition of the f_0 and is therefore essentially unknown. Making the conservative assumption $0 \leq b \leq 0.5$ (where the upper bound of 0.5 is related to the $R_b \sim 0.5$ factor in (3)) and $0^\circ \leq \vartheta \leq 360^\circ$ yields $\Delta\tilde{\phi}_s \in [-2.9^\circ, 2.8^\circ]$. This range translates into the SM range

$$S(B_s \rightarrow J/\psi f_0)|_{\text{SM}} \in [-0.086, -0.012], \quad (19)$$

while the naive value with $\Delta\tilde{\phi}_s = 0^\circ$ reads $(\sin \phi_s)|_{\text{SM}} = -0.036 \pm 0.002$ [7].

An alternative to determine the B_s^0 – \bar{B}_s^0 mixing parameters is offered by effective B_s decay lifetimes [24, 25], which are defined for a general $B_s \rightarrow f$ decay as

$$\tau_f \equiv \frac{\int_0^\infty t [\Gamma(B_s^0(t) \rightarrow f) + \Gamma(\bar{B}_s^0(t) \rightarrow f)] dt}{\int_0^\infty [\Gamma(B_s^0(t) \rightarrow f) + \Gamma(\bar{B}_s^0(t) \rightarrow f)] dt}. \quad (20)$$

The effective lifetimes of B_s decays into CP-even (such as $B_s \rightarrow K^+K^-$) and CP-odd (such as $B_s \rightarrow J/\psi f_0$) final states allow the extraction of ϕ_s and the decay width difference $\Delta\Gamma_s$. This determination is extremely robust with respect to the hadronic

penguin uncertainties, thereby nicely complementing studies of CP violation. First experimental results are already available [26], and future lifetime measurements with 1% uncertainty would be most interesting.

The current LHCb result for the extraction of ϕ_s from $B_s \rightarrow J/\psi f_0$ is given by $\phi_s = -(25 \pm 25 \pm 1)^\circ$, which corresponds to $S = -0.43^{+0.43}_{-0.34}$ [27]. In this analysis, hadronic corrections were not taken into account and are not yet relevant in view of the large experimental errors. However, once the data will enter the SM range in (19), we have to start to constrain the $\Delta\tilde{\phi}_s$. Since the hadronic effects have a different impact on $B_s^0 \rightarrow J/\psi f_0$ and $B_s^0 \rightarrow J/\psi \phi$, it will be interesting to compare the individual measurements of CP violation.

A way to obtain insights into the penguin effects is offered by $B_d^0 \rightarrow J/\psi f_0$. Its branching ratio with $f_0 \rightarrow \pi^+\pi^-$ could be as large as $\mathcal{O}(10^{-6})$ [7]. The translation of the corresponding penguin parameters into those of $B_s \rightarrow J/\psi f_0$ depends unfortunately also on assumptions about the hadronic structure of the $f_0(980)$. By the time these measurements may become available we will hopefully also have a better picture of this scalar hadronic state.

5 Conclusions

We are currently moving towards new frontiers in terms of precision. Despite the observation of a Higgs-like new particle, the LHC has not yet revealed signals of physics beyond the SM. Consequently, we have to prepare ourselves to deal with smallish NP effects, matching in particular the steadily increasing experimental precision of B -decay studies with the precision of the corresponding theoretical analyses.

In the case of the determination of the $\phi_{d,s}$ mixing phases from the benchmark decays, we are entering a territory where doubly Cabibbo-suppressed penguin contributions, which could so far be neglected, have to be controlled. The currently available data for $B_{(s)} \rightarrow J/\psi \pi, J/\psi K$ decays give $a = 0.22 \pm 0.13$, $\theta = (180.2 \pm 4.5)^\circ$ with a phase shift of $\Delta\phi_d = -(1.28 \pm 0.74)^\circ$, thereby setting the scale of the penguin effects and the associated uncertainties.

At the LHCb upgrade, the $B_s^0 \rightarrow J/\psi K_S$ decay will play an outstanding role for exploring these effects. Further insights for the measurement of ϕ_s from $B_s^0 \rightarrow J/\psi \phi$ can be obtained from $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$. In the case of the $B_{s,d} \rightarrow J/\psi f_0(980)$ system, the hadronic structure of the $f_0(980)$ affects the uncertainty of the corresponding value of ϕ_s . Future measurements of effective B_s decay lifetimes with precisions at the 1% level would offer interesting alternatives for the extraction of ϕ_s , which are very robust with respect to hadronic uncertainties.

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